THERMAL CONDUCTIVITY OF WATER *

By M. F. SOONAWALA

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ABSTRACT. Thermal conductivity of water is determined by the periodic flow method of Angstrom. A mean value of 0·00146 is obtained.

The determination of the thermal conductivities of liquids is attended with errors, the principal of which is that due to the convection currents set up. The film method surmounts this difficulty with success, the principal measurement being the quantity of heat passing through the film, for which a number of factors have to be taken into account.

The results of an experiment are reported below in which the periodic flow method is employed to obtain the conductivity of water. A heating coil, C, (Fig. 1) is made of resistance wire of about 3 ohms resistance wound as a flat spiral. This is held horizontal a few cetimetres below the surface of water in a fair-sized beaker. One junction, T, of a copper-iron thermocouple was kept projecting out of a glass tube B, the other junction dipping in water in another beaker at room temperature. The glass tube B, was attached firmly to a stand with scale and vernier and could be moved vertically up and down. The thermocouple readings were taken on a sensitive galvanometer. A current of about 1 ampere was passed steadily through the heating coil, C, for about 10 minutes, and then switched off for the same time. This was kept on to produce a periodic heat wave, which travelled down the liquid and was registered by the thermocouple. The thermocouple readings settled down to fairly even periodicity after about half a dozen periods. This was recorded as the scale deflection of the galvanometer in mms. The water was siphoned out of the beaker, the thermocouple junction, T, lowered a few mms., and the experiment repeated with the same period and heating current as before. The two periodic curves so obtained can be represented by the equations,

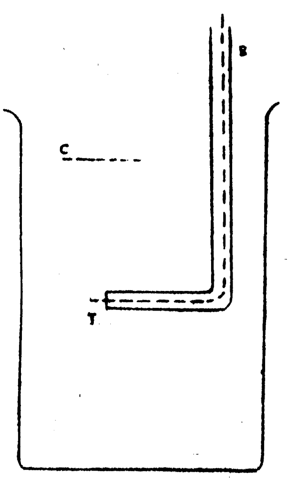

FIG. 1

$$\theta = A_0 + A_1 \sin (pt + \delta_1) + A_2 \sin (2\,pt + \delta_2) + \quad \ldots$$

and $$\theta' = A_0' + A_1' \sin (pt + \delta_1') + A_2' \sin (2\,pt + \delta_2) + \quad \ldots \quad (1)$$

θ and θ' being the temperatures at the two positions, x, and x', of the junction, T ; and $p = 2\pi/T$, where T is the period of the heat wave. Then,

$$A_1 = a_1 . e^{-\alpha_1 x} \text{ and } A_1' = a_1 . e^{-\alpha_1 x'} ; \text{ etc.} \quad \ldots \quad (2)$$

giving $$\alpha_1 \, l = \log \frac{A_1}{A_1'} \quad \ldots \quad (3)$$

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As is well known,

$$k = \frac{n\pi l^2}{T(\delta n - \delta n')\log \frac{A_1}{A_1'}} \qquad \ldots \quad (4)$$

Here $l = x - x'$, and $k = K.D/c$, where K is the thermal conductivity, D the density and c the specific heat of the liquid. The variation of temperature with time at any level is approximately simple harmonic, the slow drift of the mean value of θ being also apparent. Fig. 2 shows curves of experiment 4 of Table III. The even harmonics are absent in a flat topped curve of the kind employed, and the influence of the third harmonic is shown as an irregular depression of the curves, A and B, near their maxima and minima. A and B are the experimental curves and C a simple harmonic curve of the same period and of the amplitude of B. The curves were subjected to harmonic analysis for the determination of the amplitudes and phases. The measurement of the phase angles, δ and δ', requires a word of explanation. Readings of the galvanometer deflection in mms. recorded every minute in experiment 5 of Table III are reproduced here as Table I for the purpose. The first reading is taken the moment the current is stopped in the heating coil. The first set of readings shows that the maximum deflection is seen two minutes after the heating current is switched off; while, in the second set, the maximum follows 6 minutes after the stopping of the heating current.

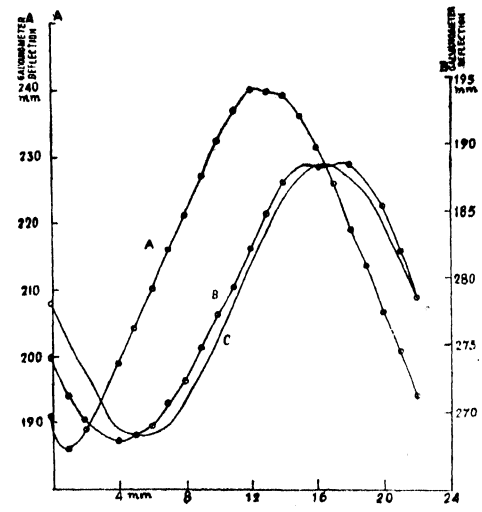

FIG. 2

TABLE I

210 mm. 214, 217, 216, 213.5, 208, 202, 194, 187, 180; 173, 169.5, 170, 175, 181, 187, 193, 200, 206, 213, 219.

182 mm., 184, 188, 190, 192, 192.5, 193, 192, 189, 185; 181, 177, 175, 173, 174, 174, 175, 177, 179.5, 183, 185.

This can be represented as follows:

In Fig. 3, A refers to the epoch of the first set, measured from the maximum, M. The angle A′ M′ is $2/20 \times 360°$, as the period is 20 min., and the angle OA′ is 54°. B represents the initial point of the second curve, measured as preceding M by 6 min., or $6/20 \times 360° = 108°$. It is the angles OA′, OB′ that are measured as phase angles 64°19′ and 7°17′ by harmonic analysis, the angles being measured with reference to O. Hence, the phase difference, $\delta - \delta'$, in this case is A′ B′, or 71°36′ Table II gives similar set of readings for experiment 4.

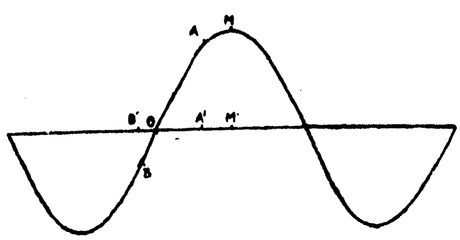

FIG. 3

TABLE II

191 mm., 186, 189, 193, 199, 204, 210 ; 216, 221, 227, 232, 237, 240, 239.5, 239, 236, 231, 225.5, 218.5, 213, 206, 200, 198.

175 mm., 172, 170, 169, 168.5, 169, 169.5, 171.5, 173, 175.5, 178; 180, 183, 186, 188, 189.3, 189, 190, 189.3, 188, 186, 182.5, 179.

Here, the phases are 1 min. and 4 min. preceding the maximum ; which correspond to angles 16° and 65° respectively. As calculated from the analysis of the curves, these are found to be 6°38′ and 63°4′. This gives a phase difference of 56°43′. It is from such considerations that $\delta-\delta'$ is calculated. The amplitudes and phase differences obtained by analysis correspond approximately to those of the curves assumed to be simple harmonic. The value of $\delta-\delta'$ is very small in experiment 2, and its result is liable to a greater error. Table III shows the results of six such experiments. Room temperature is shown as t°.

TABLE III

Exp. No.	T (min).	l (cm.)	δ.	δ'	$\delta-\delta'$.	log A_1/A_1'.	t (°c).	k.
1	20	0·90	58°56′	9°19′	68°15′	0·504	25·0	0·00154
2	24	0·50	43°7′	16°47′	26°33′	0·401	21·8	0·00129
3	18	0·80	1°53′	40°27′	38°52′	0·761	30·4	0·00157
4	22	0·80	63°4′	6°38′	56°43′	0·432	31·0	0·00156
5	20	0·80	64°19′	7°17′	71°36′	0·418	33·1	0·00140
6	24	0·80	60°17′	11°25′	71°42	0·345	33·0	0·00141

The method is free of any errors due to measurements of temperatures, temperature gradients, or quantities of heat, or radiation effects. The principal errors would be the convection currents which would be set up above levels in the liquid undergoing periodic heating and cooling. The difference of levels at which the periodic variations of temperature are investigated has necessarily to be reduced to small distances due to this effect as well as due to the fact that the amplitudes of the temperature variation becomes insensibly small for distances greater than a few millimetres. The covection effect would tend to increase the temperatures uniformly, and more so at higher levels nearer the heating coil. This would raise the mean temperature of the levels. As the mean temperature is registered by the amplitude $a_1 e^{-\alpha_1 x}$ of equation (2), this is equivalent to lowering the value of α_1 for a wave of a certain period; and, therefore, of lowering the value of log A'/A_1'. This would yield higher values of k than the real one. Convection currents, if really strong, would mix up the liquid and destroy all tendency towards regularity. As found in the experiments, the periodicity remains very pronounced even for small temperature amplitudes. The method, though not one of precision, has the merit of simplicity and directness at all stages.

DEPARTMENT OF PHYSICS;
MAHARAJA'S COLLEGE,
JAIPUR (RAJPUTANA).

REFERENCE

Preston's Theory of Heat (3rd edition), pp. 635-637.